Proposed solutions for tutorial 4

# *Intermediate Microeconomics (UTS 23567)*[*]

Preliminary and incomplete

Available at https://backwardinduction.blog/tutoring/

Office hours on Mondays from 9 am till 10 am in building 8 on level 9

Please whatsapp me on 0457871540 so I could meet you at the door, I don't have an internal phone

Also please whatsapp if you have questions, I won't be able to answer through whatsapp but I will give answer during office hours or, since you are very unlikely to have ask a unique question, in the beginning of next tutorial

Sergey V. Alexeev

10 of April, 2018

## Navigation

**Page numbers are clickable**



[*] Questions for the tutorial were provided by Massimo Scotti, slide and textbook are by Nechyba (2016). Solutions and commentary are by Sergey Alexeev (e-mail: sergei.v.alexeev@gmail.com). (Btw this is a much better book Varian (1987))

## Question 1

- Suppose that Mark has tastes described by the following utility function

$$u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

- Let's think of Good 1 as wine and of Good 2 as a composite good.
- A composite good is just an abstraction we use in economics to represent all other goods that are relevant to the consumer besides the one in question (in this case wine).
- Notice that we measure the quantity of a composite good in dollars (for example, if we say that a consumer spends $500 in all other goods we mean that he has bought an amount of "all other goods" for a value of $500).
- Since we measure the amount of a composite good in dollars, its price is always equal to $1.
- Further, assume that the price of wine is $1 per litre, and Mark's income is $12. So, summing up we have:

$$p_1 = \$1, p_2 = \$1 \text{ and } I = \$12.$$

**1.a.**

- Given the above utility function, you can check that the expression of the marginal utility of good 1 reads

$$\frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}}$$

and the expression of the marginal utility of good 2 reads

$$\frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}$$

- Find Marks demand functions for wine and other goods.

**Answer to 1.a**

Then marginal utilities of a function

$$u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

are

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{\partial x_1^{1/3} x_2^{2/3}}{\partial x_1} = \frac{1}{3} x_1^{-2/3} x_2^{2/3}$$

$$\frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{\partial x_1^{1/3} x_2^{2/3}}{\partial x_2} = \frac{2}{3} x_1^{1/3} x_2^{-1/3}$$

It follows directly from the rules of differentiation, e.g.

$$\frac{d}{dx}(cx^n) = ncx^{n-1}$$

where $c, n \in \mathbb{R}$, i.e. just numbers

$$\frac{\partial}{\partial \underbrace{x_2}_{x}} (\underbrace{x_1^{1/3}}_{c} \underbrace{x_2^{2/3}}_{x^n}) = \underbrace{\frac{2}{3}}_{n} \underbrace{x_1^{1/3}}_{c} \underbrace{x_2^{-1/3}}_{x^{n-1}}$$

where $x_1$, which otherwise is a variable, is treated as a number and $d$ is replaced with $\partial$ just to remind us about it

### Generally recall

Rules of differentiations:

$$\frac{d}{dx}(c) = 0$$
$$\frac{d}{dx}(cx) = c$$
$$\frac{d}{dx}(cx^n) = ncx^{n-1}$$
$$\frac{d}{dx}(c_1x^{n_1} + c_2x^{n_2}) = n_1c_1x^{n_1-1} + n_2c_2x^{n_2-1}$$

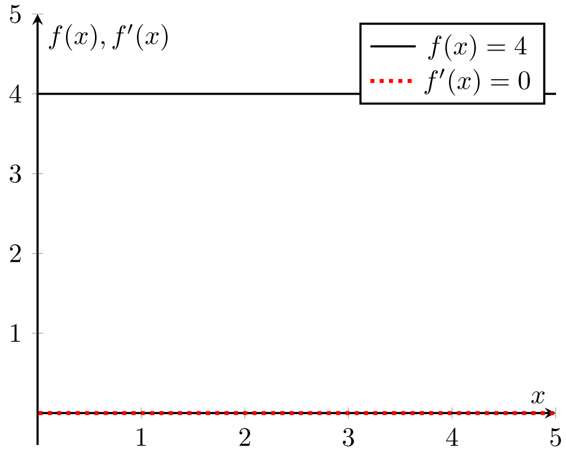
where $c, n, c_1, n_1, c_2, n_2 \in \mathbb{R}$, i.e. just numbers

Examples:

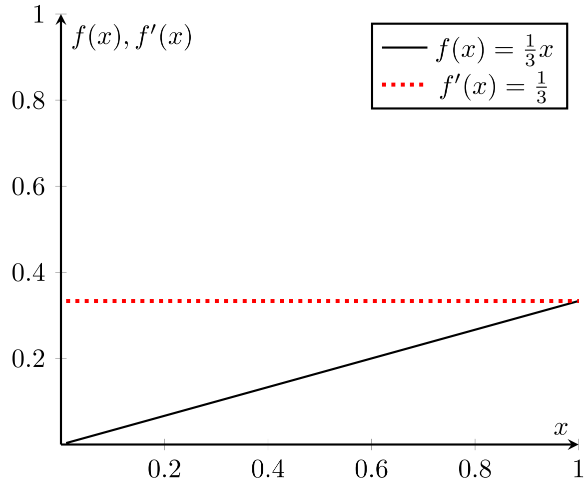
$$\frac{d}{dx}(4) = 0$$
$$\frac{d}{dx}\left(\frac{1}{3}x\right) = \frac{1}{3}$$
$$\frac{d}{dx}(2x^{1/2}) = x^{-1/2}$$
$$\frac{d}{dx}(x^3 - 3x) = 3x^2 - x$$

where in the first line $c = 4$, in the second $c = \frac{1}{3}$, in the third $c = 2, n = \frac{1}{2}$ and in the last $c_1 = 1, n_1 = 3, c_2 = -3, n_2 = 1$

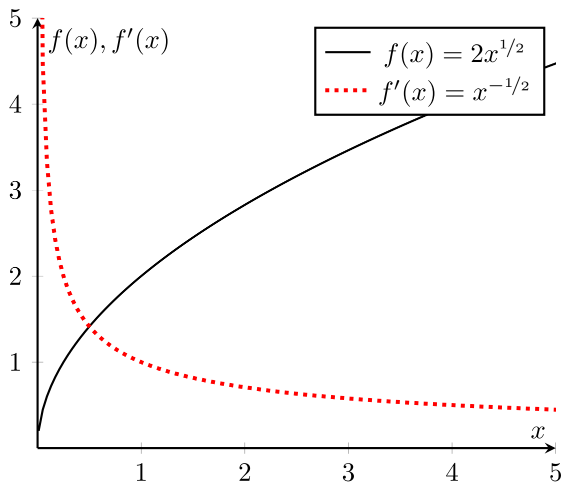
Derivative is a function itself (usually designated as $f'(x)$), which at times degenerates into a constant. Also, graphically in two dimensions a derivative is a slope, which is a line that characterizes an increase in the function's value in a small neighborhood of a function's domain.



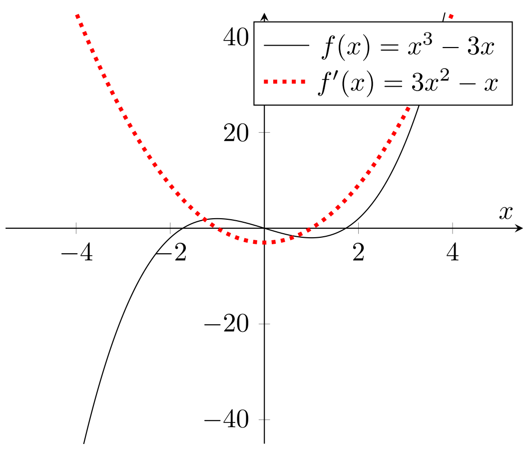

If function is 4 its slope is 0




If function is $\frac{1}{3}x$ its slope is $\frac{1}{3}$

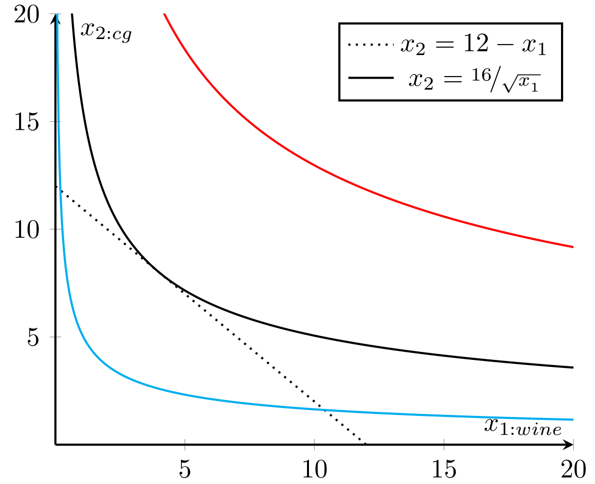


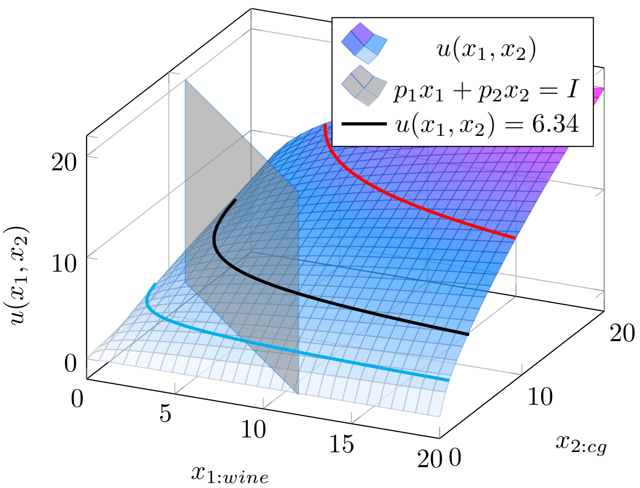

If function is $2x^{1/2}$ its slope is $x^{-1/2}$




If function is $x^3 - 3x$ its slope is $3x^2 - x$

**An optimal bundle**




Clearly an optimal bundle is characterized by

$$\boxed{MRS(x_1, x_2) = -\frac{p_1}{p_2}}$$

where the indifference curve is tangent to the budget line.

Then an optimal bundle for the function

$$u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

and parameters

$$p_1 = p_2 = 1$$

becomes

$$\begin{aligned}
MRS(x_1, x_2) &= -\frac{p_1}{p_2} \\
-\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} &= -\frac{1}{1} \\
-\frac{\frac{1}{3} x_1^{-2/3} x_2^{2/3}}{\frac{2}{3} x_1^{1/3} x_2^{-1/3}} &= -\frac{1}{1} \\
-\frac{\frac{1}{3} x_2^{1/3} x_2^{2/3}}{\frac{2}{3} x_1^{1/3} x_1^{2/3}} &= -\frac{1}{1} \\
-\frac{1}{\cancel{3}} \frac{\cancel{3}}{2} \frac{x_2^{2/3} x_2^{2/3}}{x_1^{1/3} x_1^{2/3}} &= -\frac{1}{1} \\
-\frac{x_2^{1/3+2/3}}{2 x_1^{1/3+2/3}} &= -\frac{1}{1} \\
-\frac{x_2}{2x_1} &= -\frac{1}{1} \\
\frac{x_2}{2x_1} &= 1 \\
2x_1 &= x_2
\end{aligned}$$

Whereas MRS contains information on value of $x_1$ normalized by value of $x_2$, this equation contains information on optimal ratio of products after accounting for the prices (a good might give massive increase in utility but it might be too expensive, thus, an optimal bundle corrects those already normalized values of goods for the prices). Ratio makes good sense considering that the function has a product (rather then sum) of good. Thus, a positive amount of both needs to be consumed (zero in one of good results in zero utility regardless of the amount of the other), also if $x_1 = x_2$ then $x_2^{2/3}$ gives higher utility then

$x_1^{1/3}$, thus $x_2$ has to be consumed in twice higher amounts (because $x^{2/3} = (x^{1/3})^2$). Finally, note that prices picked in way to have no effect.

**A demand function**

The demand function is a way to relate an optimal bundle to different values of prices and incomes, i.e

$$x_1(p_1, p_2, I)$$

$$x_2(p_1, p_2, I)$$

To solve for the demand function we need to combine an optimal bundle with a budget line

$$\boxed{\begin{cases} MRS(x_1, x_2) &= -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 &= I \end{cases}}$$

Then demand functions for the utility function

$$u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

parameters

$$p_1 = p_2 = 1$$

$$I = 12$$

and just found

$$MRS(x_1, x_2) = -\frac{x_2}{2x_1}$$

becomes

$$\begin{cases} MRS(x_1, x_2) &= -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 &= I \end{cases} \Rightarrow \begin{cases} -\frac{x_2}{2x_1} &= -\frac{1}{1} \\ 1x_1 + 1x_2 &= 12 \end{cases}$$
$$\Rightarrow \begin{cases} \frac{x_2}{2x_1} &= 1 \\ x_1 + x_2 &= 12 \end{cases}$$
$$\Rightarrow \begin{cases} x_2 &= 2x_1 \\ x_1 + x_2 &= 12 \end{cases}$$
$$\Rightarrow \begin{cases} x_2 &= 2x_1 \\ x_1 + 2x_1 &= 12 \end{cases}$$
$$\Rightarrow \begin{cases} x_2 &= 2x_1 \\ x_1 &= 4 \end{cases}$$
$$\Rightarrow \begin{cases} x_2 &= 8 \\ x_1 &= 4 \end{cases}$$

Note that even though a general procedure is demonstrated demand functions are very easy to see just be trying different number.

We know that an optimal bundle has twice more of $x_2$ then $x_1$, and we know that we need to spend \$12.

Assume $x_1 = 2$ then $x_2 = 4$, total is 6.

Assume $x_1 = 3$ then $x_2 = 6$, total is 9.

Assume $x_1 = 4$ then $x_2 = 8$, total is 12. Bingo!

Question 1

- Suppose that Mark has tastes described by the following utility function

$$u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

- Let's think of Good 1 as wine and of Good 2 as a composite good.
- A composite good is just an abstraction we use in economics to represent all other goods that are relevant to the consumer besides the one in question (in this case wine).
- Notice that we measure the quantity of a composite good in dollars (for example, if we say that a consumer spends $500 in all other goods we mean that he has bought an amount of "all other goods" for a value of $500).
- Since we measure the amount of a composite good in dollars, its price is always equal to $1.
- Further, assume that the price of wine is $1 per litre, and Mark's income is $12. So, summing up we have:

$$p_1 = \$1, p_2 = \$1 \text{ and } I = \$12.$$

**1.b**

- Suppose that the government introduces a 25% tax on income.
- Find the optimal bundle before and after the introduction of the tax.

**Answer to 1.b**

To solve for the demand function we need to combine an optimal bundle with a budget line

$$\boxed{\begin{cases} MRS(x_1, x_2) & = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases}}$$

Then demand functions after tax with the utility function

$$u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

parameters

$$p_1 = p_2 = 1$$

$$I = 0.75 * 12 = 9$$

where only %75 of income is preserved, and

$$MRS(x_1, x_2) = -\frac{x_2}{2x_1}$$

is

$$\begin{cases} -\frac{x_2}{2x_1} & = -\frac{1}{1} \\ 1x_1 + 1x_2 & = 9 \end{cases} \Rightarrow \begin{cases} x_2 & = 2x_1 \\ x_1 + 1x_2 & = 9 \end{cases} \Rightarrow \begin{cases} x_2 & = 6 \\ x_1 & = 3 \end{cases}$$

Again, it is fairly straightforward to see that only $x_1 = 3$ and $x_2 = 6$ satisfy two conditions that $x_2$ has to be twice higher then $x_1$ and total expenses should be equal to 9

Question 1

- Suppose that Mark has tastes described by the following utility function

$$u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

- Let's think of Good 1 as wine and of Good 2 as a composite good.
- A composite good is just an abstraction we use in economics to represent all other goods that are relevant to the consumer besides the one in question (in this case wine).
- Notice that we measure the quantity of a composite good in dollars (for example, if we say that a consumer spends \$500 in all other goods we mean that he has bought an amount of "all other goods" for a value of \$500).
- Since we measure the amount of a composite good in dollars, its price is always equal to \$1.
- Further, assume that the price of wine is \$1 per litre, and Mark's income is \$12. So, summing up we have:

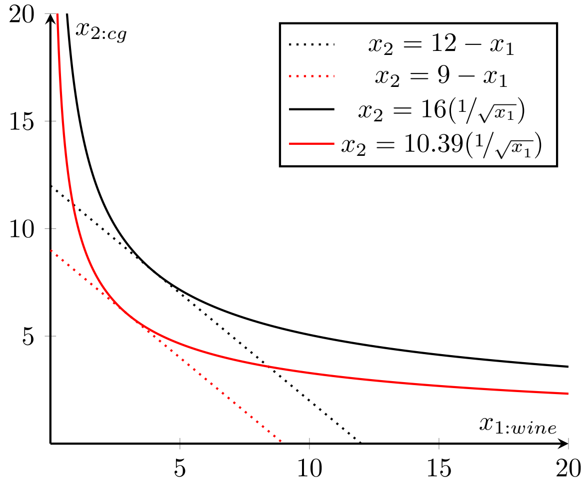
$$p_1 = \$1, p_2 = \$1 \text{ and } I = \$12.$$

**1.c**

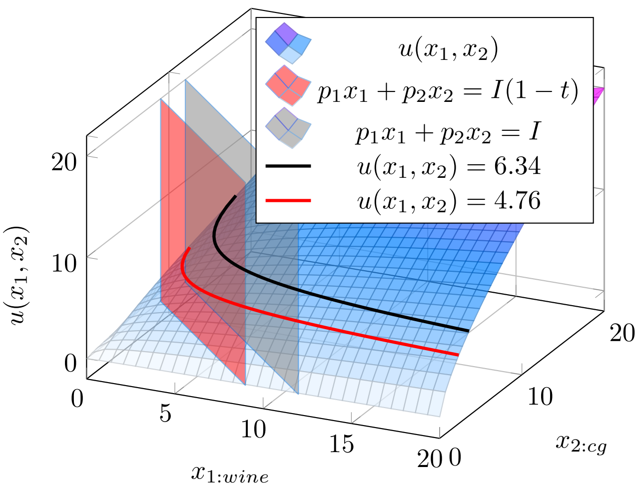
Produce two graphs.

- In one graph, represent:
  - Mark's choice set before and after the tax;
  - Mark's indifference curves;
  - Mark's optimal bundle before and after the tax.
- In the other graph, plot:
  - Mark's demand curve for wine;
  - and show how Mark's consumption of wine changes with the tax.

**Answer to 1.c**



$x_{2:cg}$
$x_{1:wine}$
$x_2 = 12 - x_1$
$x_2 = 9 - x_1$
$x_2 = 16(1/\sqrt{x_1})$
$x_2 = 10.39(1/\sqrt{x_1})$

$u(x_1, x_2)$
$p_1x_1 + p_2x_2 = I(1 - t)$
$p_1x_1 + p_2x_2 = I$
$u(x_1, x_2) = 6.34$
$u(x_1, x_2) = 4.76$
$x_{1:wine}$
$x_{2:cg}$

To plot the demand curve $x_1(p_1, p_2, I; t)$ we need to find a general form for it

$$\begin{cases} MRS(x_1,x_2) &= -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 &= I(1-t) \end{cases} \Rightarrow \begin{cases} -\frac{x_2}{2x_1} &= -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 &= I(1-t) \end{cases}$$

$$\Rightarrow \begin{cases} \frac{x_2}{2x_1} &= \frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 &= I(1-t) \end{cases}$$

$$\Rightarrow \begin{cases} x_2p_2 &= 2p_1x_1 \\ p_1x_1 + p_2x_2 &= I(1-t) \end{cases}$$

$$\Rightarrow \begin{cases} x_2 &= \frac{2p_1}{p_2}x_1 \\ p_1x_1 + p_2x_2 &= I(1-t) \end{cases}$$

$$\Rightarrow \begin{cases} x_2 &= \frac{2p_1}{p_2}x_1 \\ p_1x_1 + p_2\frac{2p_1}{p_2}x_1 &= I(1-t) \end{cases}$$

$$\Rightarrow \begin{cases} x_2 &= \frac{2p_1}{p_2}x_1 \\ p_1x_1 + \cancel{p_2}\frac{2p_1}{\cancel{p_2}}x_1 &= I(1-t) \end{cases}$$

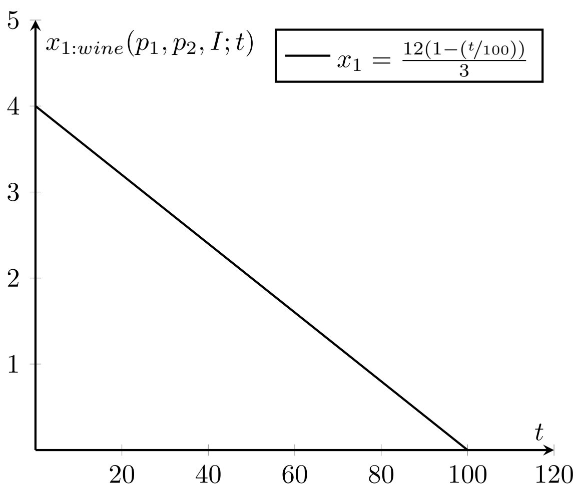
$$\Rightarrow \begin{cases} x_2 &= \frac{2p_1}{p_2}x_1 \\ p_1x_1 + 2p_1x_1 &= I(1-t) \end{cases}$$

$$\Rightarrow \begin{cases} x_2 &= \frac{2p_1}{p_2}x_1 \\ 3p_1x_1 &= I(1-t) \end{cases}$$

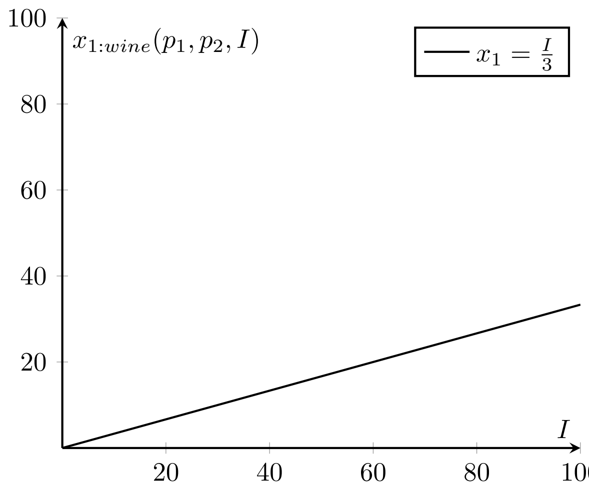
$$\Rightarrow \begin{cases} x_2 &= \frac{2p_1}{p_2}x_1 \\ x_1 &= \frac{I(1-t)}{3p_1} \end{cases}$$

$$\Rightarrow \begin{cases} x_2 &= \frac{2\cancel{p_1}}{p_2}\frac{I(1-t)}{3\cancel{p_1}} \\ x_1 &= \frac{I(1-t)}{3p_1} \end{cases}$$

$$\Rightarrow \begin{cases} x_2 &= \frac{2I(1-t)}{3p_2} \\ x_1 &= \frac{I(1-t)}{3p_1} \end{cases}$$




Evaluating the demand function at $x_1(p_1, p_2, I; t)\Big|_{(1,1,12;t)}$ allows to plot it as a function of $t$




Evaluating demand function at $x_1(p_1, p_2, I; t)\Big|_{(1,1,I;0)}$ allows to plot it as a function of $I$

Question 1 Checklist

- Suppose that Mark has tastes described by the following utility function

$$u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

- Let's think of Good 1 as wine and of Good 2 as a composite good.
- A composite good is just an abstraction we use in economics to represent all other goods that are relevant to the consumer besides the one in question (in this case wine).
- Notice that we measure the quantity of a composite good in dollars (for example, if we say that a consumer spends $500 in all other goods we mean that he has bought an amount of "all other goods" for a value of $500).
- Since we measure the amount of a composite good in dollars, its price is always equal to $1.
- Further, assume that the price of wine is $1 per litre, and Mark's income is $12. So, summing up we have:

$$p_1 = \$1, p_2 = \$1 \text{ and } I = \$12.$$

**1.a.**

- Given the above utility function, you can check that the expression of the marginal utility of good 1 reads

$$\frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}}$$

and the expression of the marginal utility of good 2 reads

$$\frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}$$

- Find Marks demand functions for wine and other goods.

**1.b**

- Suppose that the government introduces a 25% tax on income.
- Find the optimal bundle before and after the introduction of the tax.

**1.c**

Produce two graphs.

- In one graph, represent:
  - Mark's choice set before and after the tax;
  - Mark's indifference curves;
  - Mark's optimal bundle before and after the tax.
- In the other graph, plot:
  - Mark's demand curve for wine;
  - and show how Mark's consumption of wine changes with the tax.

Question 2

- Family $A$ and family $B$ have to decide how much of their monthly income to spend on *rice* and how much on *other goods*.
- Let us measure the quantity of other goods in dollar and hence set the price of "other goods" equal to \$1.
- Both families face the same economic circumstances.
  - They both have a monthly income of \$600 and face same price of rice of \$6 per kilogram.
  - Furthermore, both families live in the same country where the government has introduced a cap on the amount of rice that can be purchased monthly.
  - In particular, each family is allowed to buy a maximum of 40 kg of rice per month.
- In what follows let $x_1$ denote the quantity of rice and $x_2$ the quantity of other goods.
- Assume that the tastes of family $A$ are described by utility function

$$u_A(x_1, x_2) = x_1 x_2$$

- Where the tastes of family $B$ are described by utility function

$$u_B(x_1, x_2) = x_1^{1/4} x_2^{3/4}$$

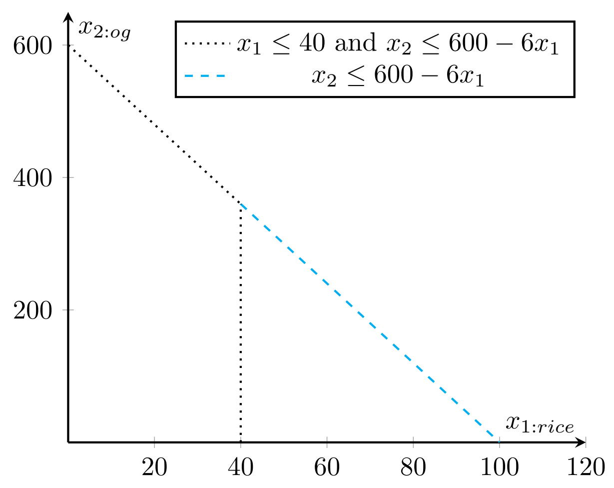
**2.a**

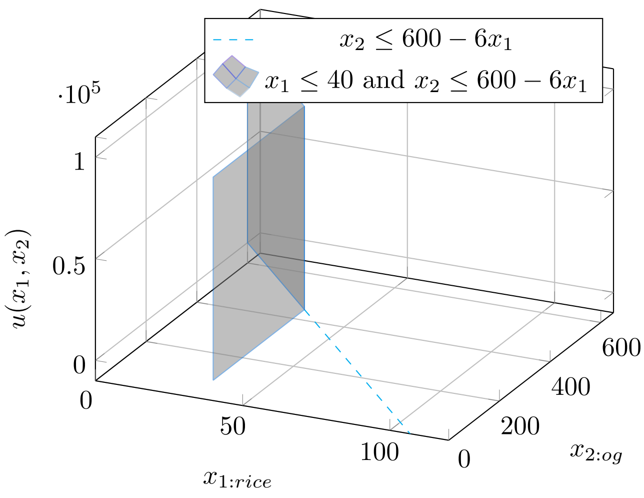
- Provide a graphical representation of the choice set of family $A$ and family $B$.
- Place $x_1$ on the horizontal axis and $x_2$ on the vertical.

**Answer to 2.a**

Budget set

$$\begin{cases} 6x_1 + x_2 & \leq 600 \\ x_1 & \leq 40 \end{cases} \Rightarrow \begin{cases} x_2 & \leq 600 - 6x_1 \\ x_1 & \leq 40 \end{cases}$$



Question 2

- Family $A$ and family $B$ have to decide how much of their monthly income to spend on *rice* and how much on other goods.
- Let us measure the quantity of other goods in dollar and hence set the price of "other goods" equal to \$1.
- Both families face the same economic circumstances.
  - They both have a monthly income of \$600 and face same price of rice of \$6 per kilogram.
  - Furthermore, both families live in the same country where the government has introduced a cap on the amount of rice that can be purchased monthly.
  - In particular, each family is allowed to buy a maximum of 40 kg of rice per month.
- In what follows let $x_1$ denote the quantity of rice and $x_2$ the quantity of other goods.
- Assume that the tastes of family $A$ are described by utility function

$$u_A(x_1, x_2) = x_1 x_2$$

- Where the tastes of family $B$ are described by utility function

$$u_B(x_1, x_2) = x_1^{1/4} x_2^{3/4}$$

**2.b**

- Given the utility function of family A, we have that for family A marginal utilities read:

$$MU_1 = x_2$$

and

$$MU_2 = x_1$$

- Find the optimal bundle chosen by family $A$.
- Show the optimal bundle on the graph above together with the indifference curve to which it belongs.

**Answer to 2.b**

Marginal utilities of a function

$$u_A(x_1, x_2) = x_1 x_2$$

are

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{\partial x_1 x_2}{\partial x_1} = x_2$$

$$\frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{\partial x_1 x_2}{\partial x_2} = x_1$$

It follows directly from the rules of differentiation, e.g.

$$\frac{d}{dx}(cx) = c$$

where $c, n \in \mathbb{R}$, i.e. just numbers

$$\frac{\partial}{\partial \underbrace{x_2}_{x}}(\underbrace{x_1}_{c}\underbrace{x_2}_{x}) = \underbrace{x_1}_{c}$$

where $x_1$, which otherwise is a variable, is treated as a number and $d$ is replaced with $\partial$ just to remind us about it

Note that

$$u_A(x_1, x_2) = x_1 x_2$$

has multiplicative nature, thus, $x_1 = 0$ or $x_2 = 0$ (boundary solutions) is never an optimal bundle

The demand functions are characterized by

$$\boxed{\begin{cases} MRS(x_1, x_2) & = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases}}$$

with parameters

$$p_1 = 6$$

$$p_2 = 1$$

$$I = 600$$

demand functions are

$$\begin{cases} MRS(x_1, x_2) & = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases} \Rightarrow \begin{cases} -\frac{\partial u(x_1,x_2)/\partial x_1}{\partial u(x_1,x_2)/\partial x_2} & = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases}$$

$$\Rightarrow \begin{cases} -\frac{x_2}{x_1} & = -\frac{6}{1} \\ 6x_1 + 1x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{x_2}{x_1} & = 6 \\ 6x_1 + x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} x_2 & = 6x_1 \\ 6x_1 + x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} x_2 & = 6x_1 \\ 6x_1 + 6x_1 & = 600 \end{cases}$$

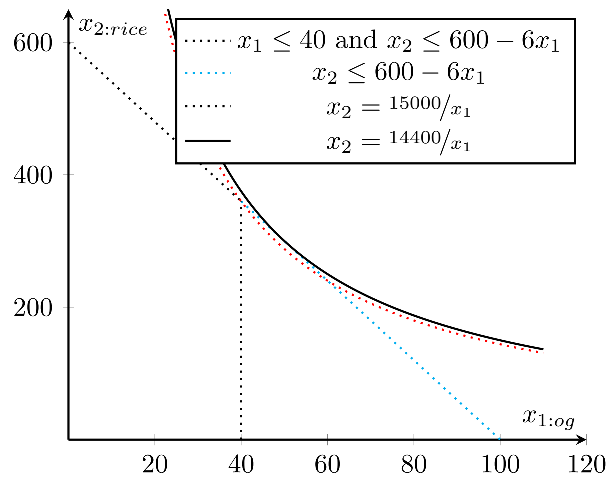
$$\Rightarrow \begin{cases} x_2 & = 6x_1 \\ 12x_1 & = 600 \end{cases}$$

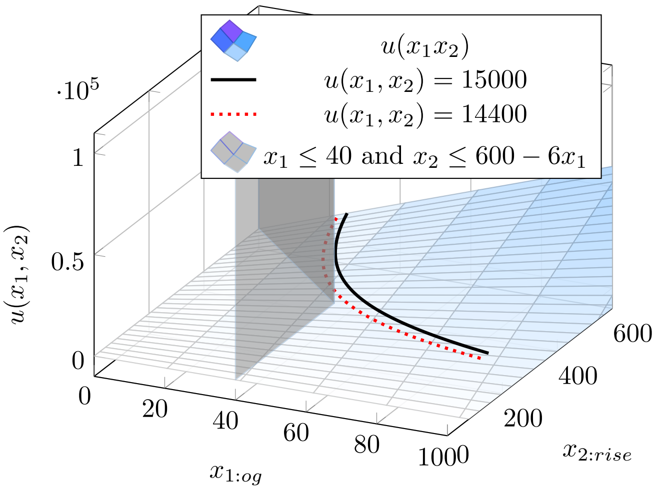
$$\Rightarrow \begin{cases} x_2 & = 6x_1 \\ x_1 & = 50 \end{cases}$$

$$\Rightarrow \begin{cases} x_2 & = 6 \times 50 \\ x_1 & = 50 \end{cases}$$

$$\Rightarrow \begin{cases} x_2 & = 300 \\ x_1 & = 50 \end{cases}$$

Note that the function is increasing in both $x_1$ and $x_2$ with equal speed (see marginal utilities). However, after adjusting for prices which have $x_2$ six times cheaper $x_1$, it is optimal to have $x_2$ six times higher then $x_1$.




A bundle

$$(50, 300)$$

even though optimal, is outside the budget set. We need to replace it with the one closest to optimal.

The maximum achievable value for good 1 is

$$x_1 = 40$$

we just need to find $x_1$ using the budget line

$$\begin{cases} x_1 & = 40 \\ 6x_1 + x_2 & = 600 \end{cases} \Rightarrow \begin{cases} x_1 & = 40 \\ 6 \times 40 + x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 & = 40 \\ x_2 & = 600 - 240 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 & = 40 \\ x_2 & = 360 \end{cases}$$

Question 2

- Family $A$ and family $B$ have to decide how much of their monthly income to spend on *rice* and how much on other goods.
- Let us measure the quantity of other goods in dollar and hence set the price of "other goods" equal to \$1.
- Both families face the same economic circumstances.
  - They both have a monthly income of \$600 and face same price of rice of \$6 per kilogram.
  - Furthermore, both families live in the same country where the government has introduced a cap on the amount of rice that can be purchased monthly.
  - In particular, each family is allowed to buy a maximum of 40 kg of rice per month.
- In what follows let $x_1$ denote the quantity of rice and $x_2$ the quantity of other goods.
- Assume that the tastes of family $A$ are described by utility function

$$u_A(x_1, x_2) = x_1 x_2$$

- Where the tastes of family $B$ are described by utility function

$$u_B(x_1, x_2) = x_1^{1/4} x_2^{3/4}$$

**2.c**

- Given the utility function of family $B$, we have that for family $B$ marginal utilities read:

$$MU_1 = \frac{1}{4} x_1^{-3/4} x_2^{3/4}$$

and

$$MU_2 = \frac{3}{4} x_1^{1/4} x_2^{-1/4}$$

- Find the optimal bundle chosen by family $B$.
- Show the optimal bundle on the graph above together with the indifference curve to which it belongs.

**Answer to 2.c**

Then marginal utilities of a function

$$u_B(x_1, x_2) = x_1^{1/4} x_2^{3/4}$$

are

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{\partial x_1^{1/4} x_2^{3/4}}{\partial x_1} = \frac{1}{4} x_1^{-3/4} x_2^{3/4}$$

$$\frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{\partial x_1^{1/4} x_2^{3/4}}{\partial x_2} = \frac{3}{4} x_1^{1/4} x_2^{-1/4}$$

It follows directly from the rules of differentiation, e.g.

$$\frac{d}{dx}(cx^n) = ncx^{n-1}$$

where $c, n \in \mathbb{R}$, i.e. just numbers

$$\frac{\partial}{\partial \underbrace{x_2}_{x}}(\underbrace{x_1^{1/4}}_{c}\underbrace{x_2^{3/4}}_{x^n}) = \underbrace{\frac{3}{4}}_{n}\underbrace{x_1^{1/4}}_{c}\underbrace{x_2^{-1/4}}_{x^{n-1}}$$

where $x_1$, which otherwise is a variable, is treated as a number and $d$ is replaced with $\partial$ just to remind us about it

The demand functions are characterized by

$$\boxed{\begin{cases} MRS(x_1, x_2) & = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases}}$$

For the utilities function

$$u_B(x_1, x_2) = x_1^{1/4} x_2^{3/4}$$

with parameters

$$p_1 = 6$$

$$p_2 = 1$$

$$I = 600$$

we have

$$\begin{cases} MRS(x_1, x_2) & = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases} \Rightarrow \begin{cases} -\frac{\partial u(x_1,x_2)/\partial x_1}{\partial u(x_1,x_2)/\partial x_2} & = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases}$$

$$\Rightarrow \begin{cases} -\frac{\frac{1}{4} x_1^{-3/4} x_2^{3/4}}{\frac{3}{4} x_1^{1/4} x_2^{-1/4}} & = -\frac{6}{1} \\ 6x_1 + 1x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} -\frac{1}{4}\frac{4}{3}\frac{x_2^{3/4} x_2^{1/4}}{x_1^{1/4} x_1^{3/4}} & = -6 \\ 6x_1 + x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} -\frac{1}{\not{4}}\frac{\not{4}}{3}\frac{x_2^{3/4+1/4}}{x_1^{1/4+3/4}} & = -6 \\ 6x_1 + x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} -\frac{x_2}{3x_1} & = -6 \\ 6x_1 + x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{x_2}{3x_1} & = 6 \\ 6x_1 + x_2 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} x_2 & = 18x_1 \\ 6x_1 + x_2 & = 600 \end{cases}$$

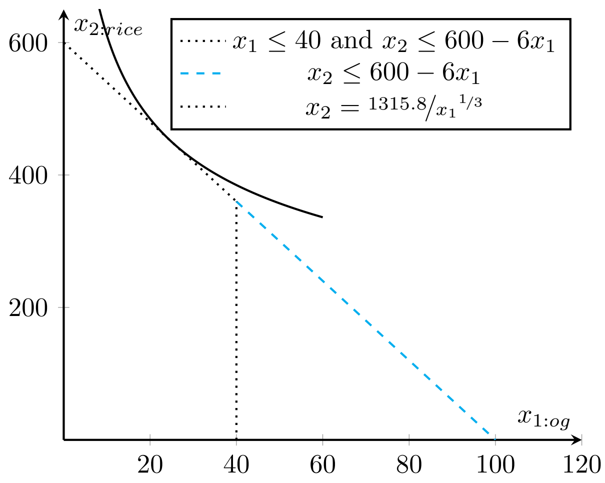
$$\Rightarrow \begin{cases} x_2 & = 18x_1 \\ 6x_1 + 18x_1 & = 600 \end{cases}$$

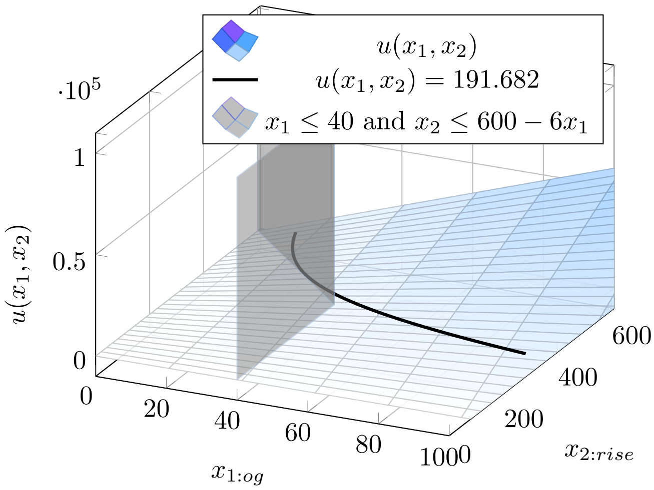
$$\Rightarrow \begin{cases} x_2 & = 18x_1 \\ 24x_1 & = 600 \end{cases}$$

$$\Rightarrow \begin{cases} x_2 & = 18x_1 \\ x_1 & = 25 \end{cases}$$

$$\Rightarrow \begin{cases} x_2 & = 18 \times 25 \\ x_1 & = 25 \end{cases}$$

$$\Rightarrow \begin{cases} x_2 & = 450 \\ x_1 & = 25 \end{cases}$$


$x_{2:rice}$
600
400
200
20
40
60
80
100
120
$x_{1:og}$
$x_1 \leq 40$ and $x_2 \leq 600 - 6x_1$
$x_2 \leq 600 - 6x_1$
$x_2 = {}^{1315.8}/{x_1}^{1/3}$


$u(x_1, x_2)$
$u(x_1, x_2) = 191.682$
$x_1 \leq 40$ and $x_2 \leq 600 - 6x_1$
$\cdot 10^5$
1
0.5
0
$u(x_1, x_2)$
0
20
40
60
80
100
0
200
400
600
$x_{1:og}$
$x_{2:rise}$

Question 2 Checklist

- Family $A$ and family $B$ have to decide how much of their monthly income to spend on *rice* and how much on *other goods*.
- Let us measure the quantity of other goods in dollar and hence set the price of "other goods" equal to \$1.
- Both families face the same economic circumstances.
  - They both have a monthly income of \$600 and face same price of rice of \$6 per kilogram.
  - Furthermore, both families live in the same country where the government has introduced a cap on the amount of rice that can be purchased monthly.
  - In particular, each family is allowed to buy a maximum of 40 kg of rice per month.
- In what follows let $x_1$ denote the quantity of rice and $x_2$ the quantity of other goods.
- Assume that the tastes of family $A$ are described by utility function

$$u_A(x_1, x_2) = x_1 x_2$$

- Where the tastes of family $B$ are described by utility function

$$u_B(x_1, x_2) = x_1^{1/4} x_2^{3/4}$$

**2.a**

- Provide a graphical representation of the choice set of family $A$ and family $B$.
- Place $x_1$ on the horizontal axis and $x_2$ on the vertical.

**2.b**

- Given the utility function of family A, we have that for family A marginal utilities read:

$$MU_1 = x_2$$

and

$$MU_2 = x_1$$

- Find the optimal bundle chosen by family $A$.
- Show the optimal bundle on the graph above together with the indifference curve to which it belongs.

**2.c**

- Given the utility function of family $B$, we have that for family $B$ marginal utilities read:

$$MU_1 = \frac{1}{4} x_1^{-3/4} x_2^{3/4}$$

and

$$MU_2 = \frac{3}{4} x_1^{1/4} x_2^{-1/4}$$

- Find the optimal bundle chosen by family $B$.
- Show the optimal bundle on the graph above together with the indifference curve to which it belongs.

Question 3

- Suppose a consumer has tastes described by the following utility function:

$$u(x_1, x_2) = 3x_1 + x_2$$

- If prices are

$$p_1 = 1$$

and

$$p_2 = 1$$

and income is

$$I = 90$$

what is the optimal bundle chosen by the consumer?

Provide a graphical representation of

- the choice set,
- indifference curves,
- and optimal bundle
- (place $x_1$ on the horizontal axis and $x_2$ on the vertical).

**Answer to 3.a**

Then marginal utilities of a function

$$u(x_1, x_2) = 3x_1 + x_2$$

are

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{\partial(3x_1 + x_2)}{\partial x_1} = 3$$

$$\frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{\partial(3x_1 + x_2)}{\partial x_2} = 1$$

It follows directly from the rules of differentiation, e.g.

$$\frac{d}{dx}(c_1 + c_2 x) = 0 + c_2$$

where $c \in \mathbb{R}$, i.e. just numbers

$$\frac{\partial}{\partial \underbrace{x_2}_{x}}(\underbrace{3x_1}_{c_1} + \underbrace{x_2}_{c_2 x}) = \underbrace{1}_{c_2}$$

where $x_1$, which otherwise is a variable, is treated as a number and $d$ is replaced with $\partial$ just to remind us about it

The demand functions are characterized by

$$\boxed{\begin{cases} MRS(x_1,x_2) & = -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 & = I \end{cases}}$$

For the utilities function

$$u(x_1,x_2) = 3x_1 + x_2$$

with parameters

$$p_1 = 1$$

$$p_2 = 1$$

$$I = 90$$

$$\begin{cases} MRS(x_1,x_2) & = -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 & = I \end{cases} \Rightarrow \begin{cases} -\frac{\partial u(x_1,x_2)/\partial x_1}{\partial u(x_1,x_2)/\partial x_2} & = -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 & = I \end{cases}$$
$$\Rightarrow \begin{cases} -3 & = -\frac{1}{1} \\ 1x_1 + 1x_2 & = 90 \end{cases}$$
$$\Rightarrow\Leftarrow$$

Must be a corner solution!

For the utilities function

$$u(x_1, x_2) = 3x_1 + x_2$$

with parameters

$$p_1 = 1$$

$$p_2 = 1$$

$$I = 90$$

You clearly need to spend all \$90 on $x_1$

Otherwise note

$$\begin{aligned} MRS(x_1, x_2) &= -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} \\ &= -\frac{3}{1} \\ &= -3 \end{aligned} \qquad \begin{aligned} -\frac{p_1}{p_2} &= -\frac{1}{1} \\ &= -1 \end{aligned}$$

You are willing to give three $x_2$ for a unit of $x_1$, whereas market exchange $x_2$ on $x_1$ on one-on-one base

Indifference curve on which the bundle $(90, 0)$ is located a collection of $x_1$ and $x_2$ that give the following level of utility

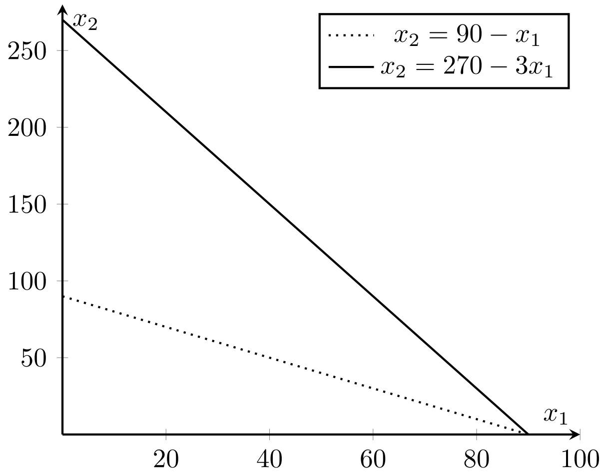
$$u(x_1, x_2)\Big|_{(90,0)} = 3(90) + (0) = 270$$

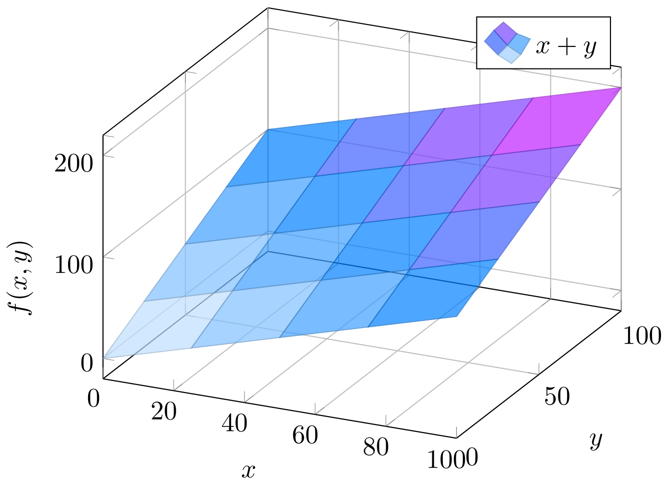
thus, an indifference curve is

$$x_2 = 270 - 3x_1$$

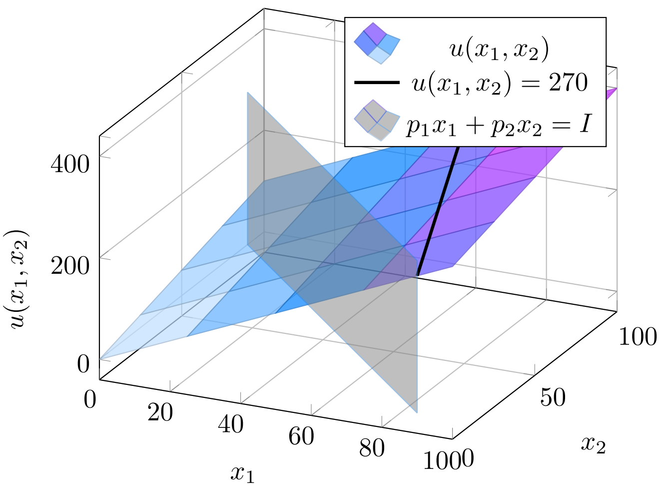
whereas a budget line as a function of $x_2$

$$x_2 = 90 - x_1$$



for a comparison



cf. the surface is tilted towards $x_1$

Question 3 Checklist

- Suppose a consumer has tastes described by the following utility function:

$$u(x_1, x_2) = 3x_1 + x_2$$

- If prices are

$$p_1 = 1$$

and

$$p_2 = 1$$

and income is

$$I = 90$$

what is the optimal bundle chosen by the consumer?

Provide a graphical representation of

- the choice set,
- indifference curves,
- and optimal bundle
- (place $x_1$ on the horizontal axis and $x_2$ on the vertical).

Question 4

Suppose a consumer has tastes described by the following utility function:

$$u(x_1, x_2) = \min(2x_1, x_2)$$

If

$$p_1 = \$3$$

and

$$p_2 = \$2$$

and income is

$$I = \$14$$

what is the optimal bundle chosen by the consumer? Provide a graphical representation of the choice set, indifference curves, and optimal bundle (place $x_1$ on the horizontal axis and $x_2$ on the vertical).

**Answer to 4**

Marginal utilities of a function

$$u(x_1, x_2) = \min(2x_1, x_2) = \begin{cases} 2x_1 & \text{if } 2x_1 < x_2 \\ x_2 & \text{if } 2x_1 > x_2 \end{cases}$$

note that function is not differentiable at

$$2x_1 = x_2$$

it is, however, differentiable at other points in the following unusual manner

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{\partial \min(2x_1, x_2)}{\partial x_1} = \begin{cases} 2 & \text{if } 2x_1 < x_2 \\ 0 & \text{if } 2x_1 > x_2 \end{cases}$$

$$\frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{\partial \min(2x_1, x_2)}{\partial x_2} = \begin{cases} 0 & \text{if } 2x_1 < x_2 \\ 1 & \text{if } 2x_1 > x_2 \end{cases}$$

Also, you don't need to know how to differentiate a function with $\min\{\cdot\}$ this is just FYI

For the utilities function

$$u(x_1, x_2) = \min(2x_1, x_2)$$

with parameters

$$p_1 = 3$$

$$p_2 = 2$$

$$I = 14$$

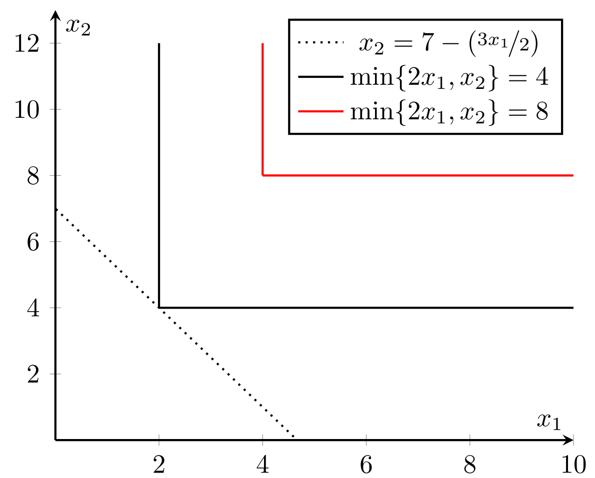
an optimal bundle always has

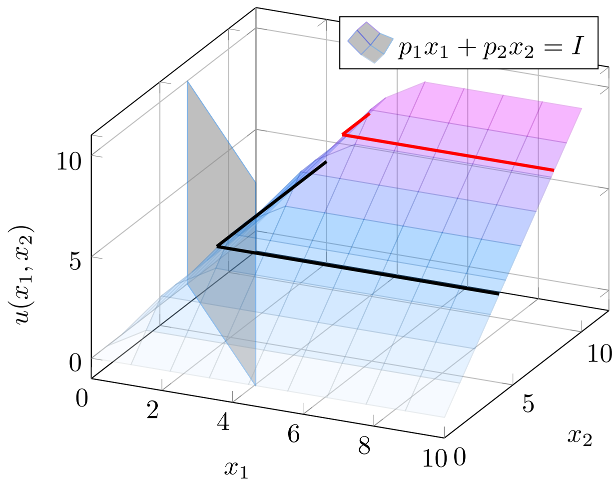
$$2x_1 = x_2$$

to find a demand functions this information has to be combined with the budget line

$$\begin{cases} 2x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= I \end{cases} \Rightarrow \begin{cases} 2x_1 &= x_2 \\ 3x_1 + 2x_2 &= 14 \end{cases}$$
$$\Rightarrow \begin{cases} 2x_1 &= x_2 \\ 3x_1 + 2(2x_1) &= 14 \end{cases}$$
$$\Rightarrow \begin{cases} x_2 &= 4 \\ x_1 &= 2 \end{cases}$$

Graphically



**Question 4 Checklist**

Suppose a consumer has tastes described by the following utility function:

$$u(x_1, x_2) = \min(2x_1, x_2)$$

If

$$p_1 = \$3$$

and

$$p_2 = \$2$$

and income is

$$I = \$14$$

what is the optimal bundle chosen by the consumer? Provide a graphical representation of the choice set, indifference curves, and optimal bundle (place $x_1$ on the horizontal axis and $x_2$ on the vertical).

## References

Nechyba, Thomas (2016). *Microeconomics: an intuitive approach with calculus*. Nelson Education. URL: https://sergeyvalexeev.files.wordpress.com/2018/03/1nechyba_t_microeconomics_an_intuitive_approach_with_calculus.pdf.

Varian, Hal R (1987). *Intermediate Microeconomics; a modern approach.* URL: https://sergeyvalexeev.files.wordpress.com/2018/04/varian-intermediate-microeconomics.pdf.